

Online MAT 1033
Scientific Notation
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Scientific notation is a way of expressing very large or extremely small numbers so that we can make these numbers more manageable. For example, in early 2015, the U.S. national debt was estimated at approximately \$18,000,000,000,000. The approximate mass of a stationary electron is 0.0000000000000000000000000000911 kg. Sources: www.informationstation.com; www.census.gov; www.physicsoftheuniverse.com; www.justfacts.com.

Writing these types of numbers using scientific notation will make them easier to read, compare, and use in computations, while avoiding the task of keeping track of all those zeros.

Scientific Notation

The scientific notation of a number is given by

$$N \times 10^k$$

where $1 \leq |N| < 10$, and k is an integer.

Note: When using scientific notation, we typically use the multiplication operator " \times ."

What does the number 5.2×10^7 , written in scientific notation, represent? We know that 10^7 equals 10,000,000. So, when we multiply the decimal part, 5.2, times 10,000,000 we get 52,000,000.

Remember from elementary mathematics that when we multiply a decimal by a power of 10 we only need to move the decimal point to the *right* as many places as there are zeros in the power. The exponent in 10^7 is $k = 7$. Since the exponent is positive, to find the previous answer without having to multiply we only need to move the decimal point 7 places to the *right* (add zeros as necessary).

$$5.2 \times 10^7 = 52,000,000$$

What would 5.2×10^{-7} represent? In this case, $k = -7$. Since the exponent is negative, we will move the decimal point 7 places to the *left* (add zeros as necessary) and we get 0.00000052.

Of course, finding the value of 5.2×10^{-7} by performing the operations we would get the same answer:

$$5.2 \times 10^{-7} = 5.2 \times \frac{1}{10^7} = \frac{5.2}{10000000} = 0.00000052$$

However, notice that it was easier to calculate 5.2×10^{-7} by just moving the decimal point 7 places to the *left* to obtain 0.00000052.

The following table summarizes the steps to convert a number written in scientific notation to the decimal notation (standard form).

Converting a Number in Scientific Notation to Standard (Decimal) Notation

- ✓ If k is positive in $N \times 10^k$, move the decimal point k places to the *right*.
- ✓ If k is negative in $N \times 10^k$, move the decimal point k places to the *left*.

Example 1

Convert 3.26×10^5 to standard notation.

Since k is positive, we will move the decimal point 5 places to the *right*: 326,000.

Optionally, you can confirm the conversion by multiplying the decimal part, 3.26, times the corresponding power of 10; that is, times 100000.

Example 2

Convert 7.659×10^{-7} to standard notation.

We notice that k is negative, thus we will move the decimal point 7 places to the *left*: 0.0000007659.

Optionally, you can confirm the conversion by multiplying the decimal part, 7.659, times the corresponding power of 10; that is, times $\frac{1}{10000000}$.

Converting a Number in Standard Notation to Scientific Notation

Write the given number in the form $N \times 10^k$, where $1 \leq |N| < 10$ and k is an integer. Note that $1 \leq |N| < 10$ means that N will be a number between 1 and 10, excluding 10.

1. Move the decimal point to the right of the first non-zero digit.
2. To find the power of 10, count the number of decimal places you moved in Step 1.
 - ✓ If the decimal point was moved left, the exponent of 10 will be positive.
 - ✓ If the decimal point was moved right, the exponent of 10 will be negative.

Example 3

Convert 137,000 to scientific notation.

First, we need to obtain a number N such that $1 \leq |N| < 10$. So, we will move the decimal point to the right of the first non-zero digit, in this case between "1" and "3." This is equivalent to moving the decimal point 5 places to the left: 137,000



The given number is greater than 10, therefore we know that the exponent k will be positive:
 $137,000 = 1.37 \times 10^5$

Example 4

Convert 0.0000000047 to scientific notation.

The given number is less than 1, thus the exponent k will be negative. To obtain a number N such that $1 \leq |N| < 10$, this time we move the decimal point between "4" and "7," or 9 places to the right: 0.0000000047

Therefore, $0.0000000047 = 4.7 \times 10^{-9}$

Example 5

In early 2015, the U.S. national debt was estimated at approximately \$18,000,000,000,000. Write this number using scientific notation.

The given number is greater than 10, therefore we know that the exponent k will be positive: We have to move the decimal point to the right of the first non-zero digit, in this case between "1" and "8." So, moving the decimal point 13 places to the left, we obtain 1.8×10^{13} .

Operations with Numbers Expressed in Scientific Notation

For the purpose of this course, we will focus on multiplication and division. We know that quantities with exponents can be multiplied and divided easily when they have the same base. All number expressed in scientific notation have base 10, thus we will use the properties of exponents to perform the computations.

Multiplication:

Multiply the two decimal parts of the numbers. Then, multiply the two powers of ten by adding the exponents.

Example 6

Multiply: $(1.37 \times 10^5)(4.7 \times 10^3)$. Express your answer using scientific notation.

$$\begin{aligned} & (1.37 \times 4.7)(10^5 \times 10^3) && \text{Regroup, then multiply.} \\ & 6.439 \times 10^8 \end{aligned}$$

Example 7

Multiply: $(1.7 \times 10^{-4})(9.25 \times 10^2)$. Write your answer using scientific notation.

$$\begin{aligned} & (1.7 \times 9.25)(10^{-4} \times 10^2) && \text{Regroup, then multiply.} \\ & 15.725 \times 10^{-2} \end{aligned}$$

Oops! Observe that the answer is not a valid scientific notation (remember we can only have 1 digit left of the decimal). Thus, we only need to move the decimal point 1 place to the left, and change the exponent of the base 10 accordingly, increasing the exponent by 1. The final answer will be:

$$1.5725 \times 10^{-1}$$

Division:

Divide the two decimal parts of the numbers. Then, divide the two powers of ten by subtracting the exponents.

Example 8

Divide: $(8.37 \times 10^5) \div (2.4 \times 10^3)$. Express your answer using scientific notation.

$$\frac{8.37 \times 10^5}{2.4 \times 10^3}$$

$$\frac{8.37}{2.4} \times \frac{10^5}{10^3} \quad \text{Regroup, then divide; subtract exponents, keeping base 10}$$

$$3.4875 \times 10^2$$

Example 9

Divide: $(5.25 \times 10^4) \div (1.5 \times 10^{-7})$. Express your answer using scientific notation.

$$\frac{5.25 \times 10^4}{1.5 \times 10^{-7}}$$

$$\frac{5.25}{1.5} \times \frac{10^4}{10^{-7}} \quad \text{Regroup, then divide.}$$

$$3.5 \times 10^{4-(-7)} \quad \text{Subtract exponents, keeping base 10}$$

$$3.5 \times 10^{11}$$

Example 10

Simplify: $\frac{(5.1 \times 10^4)(1.1 \times 10^6)}{1.5 \times 10^3}$. Express your answer using scientific notation.

$$\frac{(5.1)(1.1)}{1.5} \times \frac{(10^4)(10^6)}{10^3}$$

$$\frac{5.61}{1.5} \times \frac{10^{10}}{10^3}$$

$$3.74 \times 10^7$$

Practice Problems

Note: You will *not* turn in these problems; they will help you practice scientific notation. The answer key is included here so that you can check your work.

1. Convert to standard notation.

a. 7.26×10^8

b. 8.1×10^{-4}

2. Convert to scientific notation.

a. 56,000,000

b. 0.00000000743

3. Multiply. Express your answer using scientific notation.

a. $(2.3 \times 10^6)(4.1 \times 10^2)$

b. $(1.3 \times 10^5)(4.56 \times 10^{-8})$

c. $(6.32 \times 10^3)(4.7 \times 10^8)$

4. Divide. Express your answer using scientific notation.

a. $(7.245 \times 10^9) \div (2.3 \times 10^4)$

b. $(9.45 \times 10^{-3}) \div (4.5 \times 10^{-7})$

5. Simplify: $\frac{(2.8 \times 10^5)(3.1 \times 10^{12})}{6.2 \times 10^{11}}$. Express your answer using scientific notation.

Answer Key

1. a. 726,000,000

b. 0.00081

2. a. 5.6×10^7

b. 7.43×10^{-9}

3. a. 9.43×10^8

b. 5.928×10^{-3}

c. 2.9704×10^{12}

Note: On part (c), remember that 29.704×10^{11} is not a valid scientific notation, thus we need to move the decimal point 1 place to the left, and change the exponent of the base 10 accordingly, increasing the exponent by 1.

4. a. 3.15×10^5

b. 2.1×10^4

5. 1.4×10^6